Name:____________________________________ Date:__________________________

SOLUTION!! HW Pre Calculus 12 Section 4.4 Solving for Angles in Standard Position:

1. What is CAST? How is this acronym used in trigonometry to solve for angles? Explain:

CAST is a acronym for "COSINE, ALL, Sine, and Tangent". It is used for remembering which trig. Function will yield a positive ratio when the angles are in Q1 to Q4.
So in Q1 the Sine/cosine/tangent of all angles between 0 to 90 degrees will give a positive ratio.

In Q2, only the sine of an angle between 90 to 180 degrees will give a positive ratio. Cosine and Tangent will give a negative ratio.

Q3: only Tangent will give a positive ratio between 180 to 270 degrees. The other two functions will give a negative ratio.

Q4; Only cosine will give a positive ratio….. you get the point…..

2. Suppose $\sin\theta$ gives a negative ratio, which quadrant does θ have to be in?

   If sine of an angle gives a negative ratio, that means the "y" coordinate is negative. So the angle must be in Q3 and Q4.

3. If θ is in Quadrant 2, then which of the six trigonometric ratios are positive? Explain:

   In Q2, the Sine and Cosecant of the angle will be positive. The other four functions will give a negative ratio in Q2

4. If $\sin\theta = 0.875$, then why does the angle have to be in Quadrants 1 and 2? Explain:

   Since the sine function represents the "Y coordinate" and is it equal to 0.875. That means the Y coordinate must be positive. The y-coordinate is only positive in Q1 and Q2.

5. If $\cos\theta = -0.54$, then why does the angle have to be in Quadrants 2 and 3? Explain:
   Cosine represents the "X" coordinate. The "X" coordinate is only negative in Q2 and Q3. Simple

6. If both $\sin\theta$ and $\tan\theta$ are negative, then which quadrant will angle θ be in? Explain:

   If both sine and tangent are negative, that means cosine will be positive. Q4.

7. Suppose $\sin\theta = k$, for what value(s) of "k" will there be only ONE angle for θ ? Explain
If the "Y" coordinate is either 1 or -1, then you will have only one answer. This is because the terminal is pointing straight UP or DOWN. The angle pointing up will be 90degrees. The angle pointing down will be 270degrees.

8. Suppose $\cos^2\theta = k$, where $0 < k < 1$, how many angles θ are there? Explain:

If you square root both sides, you get $\cos\theta = \pm\sqrt{k}$ There are two values for cosine. Each value will yield two angles. Therefore, you will get FOUR answers.

9. Given that $0 \le \theta \le 2\pi$, solve for θ accurate to 3 decimal places. Show all your work and steps:

a) $3\sin\theta = 2$ $\sin\theta = \frac{2}{3}$ $\theta = 41.81°, 138.19°$	b) $4\cos\theta - 3 = 0$ $\cos\theta = \frac{3}{4}$ $\theta = 41.41°, 318.59°$	c) $\tan\theta + 4 = 0$ $\tan\theta = -4$ $\theta = -75.96°$ The angle must be between 0 and 360 $\theta = 284.03°, 104.036°$
d) $2\sin^2\theta = 1$ $\sin^2\theta = \frac{1}{2}$ $\sin\theta = \pm\sqrt{\frac{1}{2}}$ $\sin\theta = \frac{1}{\sqrt{2}}$ & $\sin\theta = -\frac{1}{\sqrt{2}}$ $\theta = 45°, 135°, 225°, 315°$	e) $-4\cos^2\theta + 5 = 2$ $-4\cos^2\theta = -3$ $\cos^2\theta = \frac{3}{4}$ $\cos\theta = \frac{\sqrt{3}}{2}$ & $\cos\theta = -\frac{\sqrt{3}}{2}$ $\theta = 30°, 140°, 210°, 330°$	f) $4\sin^2\theta + \sin\theta - 3 = 0$ This one requires factoring. It's a quadratic equation: $4A^2 + A - 3 = 0$ $(4A-3)(A+1) = 0$ $A = \frac{3}{4}, A = -1$ $\sin\theta = \frac{3}{4}, \quad \sin\theta = -1$ $\theta = 48.59°, 131.41°, 270°$

g) $6\cos^2\theta - 11\cos\theta + 4 = 0$	h) $4\cos\theta = 5\sin\theta$ Divide both sides by cosine. $4\frac{\cos\theta}{\cos\theta} = 5\frac{\sin\theta}{\cos\theta}$ $4 = 5\tan\theta$ $\frac{4}{5} = \tan\theta$ $\theta = 38.66°, 218.66°$	i) $7\cos\theta = 9\sin\theta$ $7\frac{\cos\theta}{\cos\theta} = 9\frac{\sin\theta}{\cos\theta}$ $\frac{7}{9} = \tan\theta$ $\theta = 37.87°, 217.87°$
j) $4\sin\theta\cos\theta - 3\sin\theta = 0$ For these equations, we factor out any common factor. $\sin\theta(4\cos\theta - 3) = 0$ Now solve the equation in factored form: $\sin\theta = 0 \,\&\, \cos\theta = \frac{3}{4}$ $\theta = 0°, 180°, 360°, 41.41°, 318.59°$	k) $\sin\theta = \cos\theta$ The "Y" coordinate and "x" coordinate are equal on the unit circle only when the angle is 45 degrees and 225 degrees. $\theta = 45°, 225°$	L) $\sin^2\theta\cos^2\theta - \sin^2\theta - \cos^2\theta + 1 = 0$ This one takes a little more algebra

10. For what values of θ will $\sec\theta$, $\csc\theta$, or $\cot\theta$ be undefined? Explain:

Since $\sec\theta = \frac{1}{\cos\theta}$, the function is undefined when $\cos\theta = 0$. The "X" coordinate is zero when the angle is 90degrees and 270 degrees.

For $\csc\theta = \frac{1}{\sin\theta}$, the function is undefined when $\sin\theta = 0$. The "Y" coordinate is zero when the angle is 0, 180, and 360 degrees.

For $\cot\theta = \frac{\cos\theta}{\sin\theta}$, the function is undefined when $\sin\theta = 0$. The "Y" coordinate is zero when the angle is 0, 180, and 360 degrees.

11. For what values of "k" will $\sec\theta = k$ be undefined? Explain:

To understand this one, you will need to understand how reciprocal works. $\sec\theta$ is the reciprocal of $\cos\theta$. The range of $\cos\theta$ is between 1 and -1. IT can not be bigger than 1 and smaller than -1. If you take the reciprocal of a function, the x-coordinates DON"T change, but the y-coordinates are reciprocaled. Which means, all the y-coordinates will be either "bigger or equal to 1" OR "less than or equal to -1".

To answer this question, the function $\sec\theta = k$ will be UNDEFINED when $-1 < k < 1$.

12. Given that $0 \le \theta \le 2\pi$, solve for θ. Show all your work and steps:

a) $-\frac{1}{3}\csc\theta + 1 = \frac{2}{3}$ $-\frac{1}{3}\csc\theta = \frac{2}{3} - 1$ $\frac{-1}{3}\csc\theta = \frac{-1}{3}$ $\csc\theta = 1$ $\sin\theta = 1$ $\theta = 90°$	b) $2\sec\theta = 3$ $\sec\theta = \frac{3}{2}$ $\cos\theta = \frac{2}{3}$ $\theta = 48.19°, 311.81°$	c) $3\cot\theta = 5$ $\cot\theta = \frac{5}{3}$ $\tan\theta = \frac{3}{5}$ $\theta = 30.96°, 210.96°$
d) $2\sec\theta\cot\theta = 1$ $2\left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) = 1$ $\frac{2}{\sin\theta} = 1$ $2 = \sin\theta$ No solution!!	e) $\sec\theta\sin^2\theta + \sec\theta\cos^2\theta = 3$ $\sec\theta\left(\sin^2\theta + \cos^2\theta\right) = 3$ $\sec\theta = 3$ $\cos\theta = \frac{1}{3}$ $\theta = 70.528°, 289.471°$	f) $\csc\theta\cos^2\theta + \sin\theta = 2$ $\frac{1}{\sin\theta}\cos^2\theta + \sin\theta = 2$ $\frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta} = 2$ $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta} = 2$ $\frac{1}{\sin\theta} = 2$ $\frac{1}{2} = \sin\theta$ $\theta = 30°, 150°$
g) $\csc\theta = 0$ $\frac{1}{\sin\theta} = 0$ $\frac{1}{0} = \sin\theta$ No solution! Here's another way to look at it:	h) $6\csc^2\theta - 15\csc\theta + 12$ Factor this one!!	i) $10\cot^2\theta - 29\cot\theta + 10 = 0$

$\csc\theta$ is either bigger than or equal to 1 OR smaller than or equal to -1. $\csc\theta$ can not be between -1 and 1.	$6A^2-15A+12$ $=3\left(2A^2-5A+4\right)$ $A=\dfrac{5\pm\sqrt{25-4(2)4}}{4}$ $A=\dfrac{5\pm\sqrt{-7}}{4}$ Crap…ooops… no solution…	